Qual Syllabus

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1   Truth and Realism

In his essay "Truth", Michael Dummett points out that an ability to categorize sentences as true or false does not give a complete understanding of the concept without knowledge of what role the concept plays in our lives.

McDowell and Davidson suggest that its role is to support a theory of meaning that allows us to say what a speaker means through her use of language. It plays a role as a constraint that may tell us when a putative translation of her sentences is incorrect. Field, on the other hand, suggests that its more important role is in explaining meaning by demonstrating the connections between words and the world.

In either case, the concept of truth plays an important role in the theory of meaning. In *The Logical Basis of Metaphysics* Dummett suggests that the best way to approach fundamental questions about the nature of reality is through understanding the meanings of our sentences that purport to answer such questions. He seeks to characterize anti-realist and realist metaphysical positions in terms of both their theories of meaning and their logic. To what extent do these characterizations agree, and are they justified?

Given such a theory of meaning, how does the notion of truth emerge? Is such a notion of truth primary, secondary, or irrelevant to a theory of logical inference? Paul Horwich thinks that the theory of truth can shed little or no light on this or any other issue. To what extent is he right, and to what extent is he missing the relevant point?

Having translated the metaphysical debate at least partially into linguistic and logical terms, Dummett argues in favor of a sort of anti-realism. Why does he favor this sort of anti-realism, and do his arguments really suffice ot establish an intuitionist logic, or is Crispin Wright correct in saying that an anti-realist could be a classical logician?

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2 Mathematical Nominalism

In *Science Without Numbers*, Hartry Field puts forward a modern version of the theory that mathematical objects don't literally exist, and that therefore mathematics is literally false. He starts by making some epistemic objections (later refined by Øystein Linnebo) that raise troubles for a platonist, and then goes on to describe a scientific practice that makes no use of mathematical objects, and therefore avoids the Quine-Putnam indispensibility argument. He explains that since the platonist theory is a "conservative extension" of the nominalist theory, it is therefore usable for deriving nominalist consequences, even if one believes it to be literally false.

The first question one might ask is just what the epistemic problems for platonism are supposed to be, and why they matter, if Field has really shown that we can do without mathematical entities?

The second question is whether his program really succeeds as he claims at nominalizing physics? The argument that mathematics is conservative seems to rely either on platonist assumptions or some unjustified claims that have been used naively by platonists in past. But Stewart Shapiro gives some evidence that if Field is using a first-order logical theory, then mathematics really isn't conservative.

In order to respond to Shapiro's point, Field may need to use some sort of higher-order logic, but it's not clear just what sort of logic is available for his theory. A full second-order theory seems to rely on nominalistically unacceptable set theory, and a pure first-order theory seems to weak. Field explicitly relies on some sort of logic of mereology, but it's not clear just how to formulate this.

In addition, while Field has only managed to nominalize a Newtonian physical theory, he has faced even greater difficulties in nominalizing the logical theories that he needs to use to carry out any sort of reasoning at all. What sorts of modal resources does Field help himself to in discussing logic, and how does he make use of them without reference either to abstract models of theories or abstract deductions of formulas?

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3 Set Theory

In the wake of Gödel's incompleteness theorems, Hilbert's program to build up all of mathematics by deduction from the axioms was abandoned. For a while, mathematicians largely retreated to working with the consequences of some particularly trusted theory or other (Russellian type theory, PA, or ZFC). But in 1947, Gödel suggested a new way forward, stating his opinion that unsolved problems like CH could be solved by appeal to new axioms, justified largely because of their "abundant ... verifiable consequences", suggesting large cardinal axioms as one such class of candidates.

Gödel's program was motivated by his particular brand of platonism, and Solomon Feferman seems to agree that this is the only sort of justification possible for such a program. However, Penelope Maddy seems to disagree. Which, if either, of them is right about what philosophical position one must take in order to go beyond the axioms?

It seems plausible to think that no matter what one's philosophical position is, there is a substantive question here. Martin has phrased the new question as inquiring as to what counts as "mathematically proper evidence" for a proposition. Just what does it take for evidence to be "mathematically proper", and how does it differ from the evidence that is offered in other natural sciences?

Once one has decided what evidence is mathematically proper, it seems plausible to ask just which propositions can be decided by this addition evidence beyond pure deduction from the axioms, if any. Many, like Feferman, have argued that the Continuum Hypothesis will be forever out of this range, but others have hope that it may be solved by a process similar to this. If it can't be solved by these methods, what are the implications for set theory? Does it mean that a notion as simple as that of "set of natural numbers" is ambiguous, and if so, how?

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